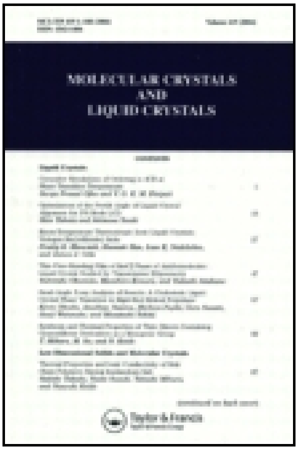

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Isothermal Hydrodynamics of Biaxial Nematics

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Isothermal Hydrodynamics of Biaxial Nematics

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Differential equations governing elastic and flow behavior of compressible biaxial nematics are derived using the Ericksen–Leslie approach (ELA). Expressions of the free energy density F, viscous stress σ'_{ji} and flexoelectric polarisation $P_i^{(f)}$ are derived for monoclinic and triclinic nematics using the principal axes approach utilised by Saupe. The expression of σ'_{ji} derived using ELA can be reduced to that derived from Saupe's approach by adopting a procedure which was developed in an earlier communication. For biaxial nematics, the number of elastic constants seems to be the same as the number of viscosity coefficients, in general.

1. INTRODUCTION

Of the many theories which have been proposed for describing the hydrodynamic behaviour of biaxial nematics,[1] the one given by Saupe[2] for a compressible orthorhombic nematic comes closest in approach to the Ericksen–Leslie theory of uniaxial nematics. Genaralising the elastic theory of crystals, Saupe shows[2] that an orthorhombic nematic is described by a bulk modulus, 12 elastic constants and 15 viscosity coefficients. More recently the Ericksen–Leslie approach (ELA) has been extended[3] to describe orthorhombic nematics. It has been shown[3] that the expression for viscous stress σ'_{ji} derived from ELA becomes formally identical to that derived by Saupe[2] when the dissipative function approach (DFA) is utilised.

While the orthorhombic is by far the most symmetrical class of biaxial symmetry, the existence of the more complex or asymmetric classes of nematics, viz. the monoclinic and triclinic cannot be ruled out (see for intance the theoretical prediction in ref. 4 and discovery of a complex orientation in ref. 5). In this communication, expressions for elastic free energy density F, σ'_{ji} and flexoelectric polarisation $P_i^{(f)}$ have been derived for triclinic and monoclinic nematics on the basis of the principal axes method used by Saupe.[2] Extending the procedure adopted in ref. 3 the expression for σ'_{ji} derived from ELA is shown to reduce to the corresponding expression derived from Saupe's approach, for both monoclinic and triclinic classes. Expressions of F and σ'_{ji} for a compressible triclinic or monoclinic nematic go over to those given by Saupe[2] when orthorhombic symmetry is assumed for the director field. For a given class of biaxial symmetry, the number (n_{vis}) of viscosity coefficients of a compressible fluid is found to equal the number of elastic constants (n_{el}) plus the number of surface terms; for an incompressible fluid, $n_{\text{vis}} = n_{\text{el}}$. As stated earlier, there are other hydrodynamical theories which have been presented for biaxial nematics.[6–12] However a discussion of these falls outside the scope of the present paper.

2. CONSERVATION LAWS, CONSTITUTITIVE ASSUMPTIONS AND ENTROPY INEQUALITY

The approach adopted in this communication closely follows that used in ref. 3. Let the preferred direction of molecular orientation of a compressible biaxial nematic be represented at each point x_k by a non-coplanar triad of unit vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ whose magnitudes and projections along one another remain constant ($|\mathbf{a}|^2 = |\mathbf{b}|^2 = |\mathbf{c}|^2 = 1$; $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{b} \cdot \mathbf{c}$ and $\mathbf{c} \cdot \mathbf{a}$ are constant). Temperature T is also assumed to be fixed. Following ref. 3, it is straightforward to write down the following relations

$$\dot{\rho} + \rho v_{k,k} = 0 \tag{1}$$

$$\rho \dot{v}_i = \sigma_{ji,j} + \rho f_i \tag{2}$$

$$\partial F/\partial d_{ij} = 0; \qquad \partial F/\partial N_i^a = 0 \qquad \text{(CP)} \tag{3}$$

$$\sum_a \left[a_k \partial F/\partial a_i + a_{k,j}\, \partial F/\partial a_{i,j} + a_{j,k}\, \partial F/\partial a_{j,i} \right]$$

$$= \sum_a \left[a_i\, \partial F/\partial a_k + a_{i,j}\, \partial F/\partial a_{k,j} + a_{j,i}\, \partial F/\partial a_{j,k} \right] \tag{4}$$

$$\sigma_{ji} = \sigma_{ji}^0 + \sigma'_{ji}\,, \qquad g_i^a = g_i^{a0} + g_i^{a'} \qquad \text{(CP)},$$

$$\sigma_{ji}^{0} = -\rho^{2}\delta_{ij}\,\partial F/\partial\rho - \rho\sum_{a} a_{k,i}\,\partial F/\partial a_{k,j},$$

$$g_{i}^{a0} = -\rho\,\partial F/\partial a_{i}, \qquad \pi_{ji}^{a} = \rho\,\partial F/\partial a_{i,j} \qquad \text{(CP)} \tag{5}$$

$$\rho T\dot{S} = \sigma'_{ji}d_{ij} - \sum_{a} g_{i}^{a'}N_{i}^{a} \geqslant 0 \tag{6}$$

$$\sigma'_{ki} - \sigma'_{ik} = \sum_{a}\left[g_{i}^{a'}a_{k} - g_{k}^{a'}a_{i}\right] = \sum_{a}\Big[a_{k}\big(\rho\rho_{a}\ddot{a}_{i} - \pi_{ji,j}^{a} - g_{i}^{a0} - \rho G_{i}^{a}\big) - a_{i}\big(\rho\rho_{a}\ddot{a}_{k} - \pi_{jk,j}^{a} - g_{k}^{a0} - \rho G_{k}^{a}\big)\Big] \tag{7}$$

Notation is as in ref. 3. In particular the N_i^a (CP) are not all independent, but are related to **N** the rotational velocity of the director field relative to the fluid (see Appendix 2). The external director body forces G_i^a (CP) are generally produced by the use of an external field which interacts with the system via the respective susceptibility. In the case of an electric field it may be necessary to consider flexoelectric effects.[13] This has been treated in subsequent sections. F is a function of ρ, a_i, $a_{j,k}$ (CP) while σ'_{ji} and $g_i^{a'}$ (CP) are functions of d_{ij} and N_i^a (CP). Eqs. (2) and (7) should help determine the three components of velocity and three quantities (say angles) which describe the director field.

3. F, σ'_{ji} AND $P_i^{(f)}$ FOR THE TRICLINIC CLASS

A. Saupe's approach

Let the director field of a uniformly aligned triclinic nematic be described by unit vectors **a**, **b**, **c** which are non-coplanar but not necessarily mutually orthogonal. Without loss of generality, the coordinate axes can be chosen such that x_1 is along **a**, and **b** lies in the x_1x_2 plane. Then the aligned director configuration is

$$\mathbf{a} = (1,0,0), \qquad \mathbf{b} = (d_1, d_2, 0), \qquad \mathbf{c} = (e_1, e_2, e_3),$$

$$d_1^2 + d_2^2 = 1 = e_1^2 + e_2^2 + e_3^2 \qquad d_1 = \mathbf{a}\cdot\mathbf{b}, \tag{8}$$

$$e_1 = \mathbf{c}\cdot\mathbf{a}, \qquad e_1d_1 + e_2d_2 = \mathbf{b}\cdot\mathbf{c}, \qquad e_3d_2 = \mathbf{a}\cdot\mathbf{b}\times\mathbf{c}$$

Under small rotations α_i about the axes x_i, the director field is distorted so that

$$\mathbf{a} = (1, \alpha_3, -\alpha_2), \qquad \mathbf{b} = (d_1 - d_2\alpha_3, d_2 + d_1\alpha_3, d_2\alpha_1 - d_1\alpha_2),$$
$$\mathbf{c} = (e_1 - e_2\alpha_3 + e_3\alpha_2, e_2 + e_1\alpha_3 - e_3\alpha_1, e_3 + e_2\alpha_1 - e_1\alpha_2) \tag{9}$$

where α_i are functions of x_k. The torsion tensor $\alpha_{lm} = \alpha_{l,m}$ is in generally asymmetrical. Following ref. 2, F is written as a quadratic form in the principal axes frame

$$\rho F = F_0 + \lambda_{ijkl} u_{ij} u_{kl}/2 + \mu_{ijkl}\alpha_{ij}\alpha_{kl}/2 + [\,\mu_{ik}\alpha_{ik} + \kappa_{iklm} u_{ik}\alpha_{lm}] \tag{10}$$

where $u_{ij} = (u_{i,j} + u_{j,i})/2$ is the familiar symmetrical strain tensor corresponding to a linear displacement u_i. The terms [ ] exist only for an enantiomorphic system and vanish for a nematic. The only symmetry imposed on μ_{ijkl} is that $\mu_{pqrs} = \mu_{rspq}$, so that their number is 45. As the system is a fluid, $\lambda_{ijkl} u_{ij} u_{kl} \to \lambda(u_{jj})^2$ and $\kappa_{iklm} u_{ik}\alpha_{lm} \to \kappa_{11lm}(u_{jj})\alpha_{lm}$. Eq. (10) can now be written as

$$\rho F = F_0 + \frac{\lambda}{2}\left(\sum_j u_{jj}\right)^2 + \frac{1}{2}\sum_{\substack{i=k\\ j=l}} \mu_{ijkl}\alpha_{ij}\alpha_{kl} + \sum_{\substack{i\neq k\\ j\neq l}} \mu_{ijkl}\alpha_{ij}\alpha_{kl}$$
$$+ \left[\sum_{i,k}\left(\mu_{ik} + \kappa_{11ik}\sum_j u_{jj}\right)\alpha_{ik}\right] \tag{11}$$

This expression can be cast into a frame indifferent form by transformations (see Appendix 3) which show that each α_{ij} can be written as a linear combination of one or more of the nine quantities

$$I_{abc} = a_i b_{k,i} c_k, \qquad I_{aab} = a_i a_{k,i} b_k, \qquad I_{aac} = a_i a_{k,i} c_k \qquad \text{(CP)} \tag{12}$$

The [ ] terms result in nine linear terms while $\mu_{ijkl}\alpha_{ij}\alpha_{kl}$ contributes 45 second order terms each of whose coefficients is a linear combination of the μ_{ijkl}. Now using the expression for the triclinic metric (see Appendix 1) and following Ericksen,[14] six divergence terms can be constructed

$$
\begin{aligned}
D_{aa} &\equiv (a_l a_{k,l} - a_k a_{j,j})_{,k} \\
&= 2\big[(I_{aab}I_{bca} - I_{aac}I_{bba})a_c - a_b(I_{cca}I_{aab} + I_{aac}I_{cab}) \\
&\qquad - a_a(I_{bca}I_{cab} + I_{bba}I_{cca})\big]/(\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}), \\
D_{ab} &\equiv (a_l b_{k,l} - a_k b_{j,j})_{,k} \\
&= \big[b_c(I_{aab}I_{bca} - I_{aac}I_{bba}) - c_a(I_{aab}I_{bbc} + I_{bba}I_{abc}) \\
&\qquad - a_b(I_{bca}I_{cab} + I_{cab}I_{abc} + I_{bba}I_{cca} + I_{ccb}I_{aab}) \\
&\qquad - a_a(I_{cca}I_{aab} + I_{aac}I_{cab}) \\
&\qquad + b_b(I_{bbc}I_{cab} - I_{bba}I_{ccb})\big]/(\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}) \quad \text{(CP)} \qquad (13)
\end{aligned}
$$

From Eq. (13) it is clear that some six of the products $I_{uvw}I_{xyz}$ occurring therein, can be written in terms of the six divergence terms and the remaining I products. There is clearly more than one way of doing this. However, keeping in mind the expression of F for orthorhombic nematic,[2] the six products $I_{abc}I_{bca}, I_{aac}I_{cab}$ (CP) are eliminated from the expression for F, which can now be written as

$$
\begin{aligned}
\rho F = F_0 &+ \lambda(u_{ii})^2/2 \\
&+ \Big\{\sum_a \big[(k_a + \kappa_a u_{jj})I_{abc} + (l_a + \lambda_a u_{jj})I_{aab} + (m_a + \mu_a u_{jj})I_{bba}\big]\Big\} \\
&+ \sum_a \big[k_{aa}I_{abc}^2 + k_{ab}I_{aab}^2 + k_{ac}I_{aac}^2 + 2c_{ab}I_{aac}I_{bbc} + 2c_{aa}I_{aab}I_{aac} \\
&\qquad + 2c_{ba}I_{aab}I_{abc} + 2l_{aa}I_{aab}I_{bba} + 2l_{ab}I_{aab}I_{bbc} + 2l_{ba}I_{aab}I_{bca} \\
&\qquad + 2m_{aa}I_{aab}I_{cab} + 2m_{ab}I_{aac}I_{abc} + 2m_{ba}I_{aac}I_{bba} \\
&\qquad + 2p_{aa}I_{aac}I_{bca} + 2k_{0a}D_{aa} + 2l_{0a}D_{ab}\big]/2 \qquad (14)
\end{aligned}
$$

The terms { } will not occur for a nematic. The different elastic constants k, l, m, are linear combinations of the components of the tensor μ, with coefficients which depend upon the scalar products $\mathbf{a}\cdot\mathbf{b}$, $\mathbf{b}\cdot\mathbf{c}$, $\mathbf{c}\cdot\mathbf{a}$ and $\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}$. As these expressions are cumbersome, they have not been presented. Eq. (14) shows that F for a triclinic nematic depends on 39 elastic constants and 6 surface terms. It has been stated[15] that the number of independent elastic constants of a triclinic crystal can be reduced from 21 to 18 by including three angles in the description. This possibility has not been explored in this communication. Substitution of Eq. (14) shows that F satisfies Eq. (4) identically.

The expression for σ'_{ji} the viscous stress is derived following refs. 2 and 3. In the principal axes frame, the dissipative function $\varphi = \rho T\dot{S}/2$ is written as a quadratic form in d_{ij} (symmetric part of the velocity gradient $v_{i,j}$) and $\dot{\alpha}_i - \dot{\beta}_i$ the velocity of rotation of the director field relative to the fluid (which represents the antisymmetric part of $v_{i,j}$)

$$\dot{\beta}_i = \epsilon_{ikl} v_{l,k}/2, \qquad \omega_{mj} = \epsilon_{jmi}\dot{\beta}_i,$$
$$\varphi = \left[\eta_{ijkl} d_{ij} d_{kl} + \gamma^{(1)}_{ik}(\dot{\alpha}_i - \dot{\beta}_i)(\dot{\alpha}_k - \dot{\beta}_k) + \gamma^{(2)}_{ikl}(\dot{\alpha}_i - \dot{\beta}_i) d_{kl} \right]/2 \tag{15}$$

As the only symmetry to be satisfied by the tensors is mathematical, η_{ijkl} contributes 21 independent coefficients, $\gamma^{(1)}_{ik}$ six and $\gamma^{(2)}_{ikl}$ 18, so that the compressible triclinic nematic has 45 viscosity coefficients. *This brings out an interesting fact that in* DFA, *the viscous stress and the dissipative function are described by the same number of viscosity coefficients as F is by elastic constants and surface terms*. In a way it is not surprising, because the quadratic forms written for F and φ are essentially similar for a biaxial nematic. While the expression for F is built out of nine curvature strains I_{xyz}, the expression for φ is constructed from the nine quantities which represent the symmetric and antisymmetric parts of the velocity gradient. In frame independent notation, (see Appendix 3) the dissipative function can be written as

$$\begin{aligned} 2\varphi = \sum_a \Big[&\eta_{aaaa} A_a^2 + 2\eta_{aabb} A_a B_b + 4\eta_{aaab} A_a A_b + 4\eta_{aaca} A_a A_c \\ &+ 4\eta_{aabc} A_a B_c + 4\eta_{abab} A_b^2 + 8\eta_{abca} A_b A_c + \gamma_{aaa} N_l a_l A_a \\ &+ \gamma_{baa} N_l b_l A_a + \gamma_{caa} N_l c_1 A_a + 2\gamma_{aab} N_l a_l A_b + 2\gamma_{bab} N_l b_l A_b \\ &+ 2\gamma_{cab} N_l c_l A_b + \gamma_{aa}(N_l a_l)^2 + 2\gamma_{ab} N_l a_l N_m b_m \Big] \end{aligned} \tag{16}$$

where $A_a = a_l d_{lk} a_k$, $A_b = B_a = a_l d_{lk} b_k$ (CP). The viscosity coefficients in Eq. (16) have been weighted keeping in mind the expression of φ for an orthorhombic nematic[2] and are linear combinations of the components of the viscosity tensors η and γ. As the expressions are somewhat lengthly, they have not been presented here. Following ref. 2 and ref. 3, the viscous stress σ'_{ji} is defined as

$$\sigma'_{ji} \equiv \sigma'^{s}_{ji} + \sigma'^{a}_{ji} = \partial\varphi/\partial d_{ij} - \partial\varphi/\partial\Omega_{ij} \tag{17}$$

where Ω_{ij} is the antisymmetrical tensor which represents **N** (Appendix 2). The definition of σ'^{a}_{ji} given in Eq. (17) is general and can cover cases where the director field undergoes a dissipative relaxation without the occurrence of material flow. From Eqs. (16) and (17) the viscous stress can be written as

$$\begin{aligned}
\sigma'_{ji} = \sum_a \big[& a_i a_j (2\eta_{aaaa} A_a + 4\eta_{aaab} A_b + 4\eta_{aaca} A_c + 2\eta_{aabb} B_b + 4\eta_{aabc} B_c \\
& + 2\eta_{ccaa} C_c + \gamma_{aaa} N_l a_l + \gamma_{baa} N_l b_l + \gamma_{caa} N_l c_l) \\
& + a_j b_i (\eta^{-}_{aaab} A_a + \eta^{-}_{abab} A_b + \eta^{-}_{caab} A_c + \eta^{-}_{bbab} B_b + \eta^{-}_{abbc} B_c \\
& + \eta^{-}_{ccab} C_c + \gamma^{-}_{aab} N_l a_l + \gamma^{-}_{bab} N_l b_l + \gamma^{-}_{cab} N_l c_l) \\
& + a_i b_j (\eta^{+}_{aaab} A_a + \eta^{+}_{abab} A_b + \eta^{+}_{caab} A_c + \eta^{+}_{bbab} B_b + \eta^{+}_{abbc} B_c \\
& + \eta^{+}_{ccab} C_c + \gamma^{+}_{aab} N_l a_l + \gamma^{+}_{bab} N_l b_l + \gamma^{+}_{cab} N_l c_l) \big]/2;
\end{aligned}$$

$$\begin{aligned}
\eta^{\pm}_{aaab} &= 2\eta_{aaab} \pm (\gamma_{aaa} c_a + \gamma_{baa} c_b + \gamma_{caa} c_c)/2 \\
\eta^{\pm}_{abab} &= 4\eta_{abab} \pm (\gamma_{aab} c_a + \gamma_{bab} c_b + \gamma_{cab} c_c) \\
\eta^{\pm}_{caab} &= 4\eta_{caab} \pm (\gamma_{aca} c_a + \gamma_{bca} c_b + \gamma_{cca} c_c) \\
\eta^{\pm}_{bbab} &= 2\eta_{bbab} \pm (\gamma_{abb} c_a + \gamma_{bbb} c_b + \gamma_{cbb} c_c)/2 \\
\eta^{\pm}_{abbc} &= 4\eta_{abbc} \pm (\gamma_{abc} c_a + \gamma_{bbc} c_b + \gamma_{cbc} c_c) \\
\eta^{\pm}_{ccab} &= 2\eta_{ccab} \pm (\gamma_{acc} c_a + \gamma_{bcc} c_b + \gamma_{ccc} c_c)/2 \\
\gamma^{\pm}_{aab} &= \gamma_{aab} \pm (\gamma_{aa} c_a + \gamma_{ab} c_b + \gamma_{ca} c_c), \\
\gamma^{\pm}_{bab} &= \gamma_{bab} \pm (\gamma_{ab} c_a + \gamma_{bb} c_b + \gamma_{bc} c_c), \\
\gamma^{\pm}_{cab} &= \gamma_{cab} \pm (\gamma_{ca} c_a + \gamma_{bc} c_b + \gamma_{cc} c_c)
\end{aligned} \tag{18}$$

Using Eqs. (18), (14) and (7) it should be possible to write down the torque equation in the absence of external fields.

If the material is flexoelectric, electric polarisation $P_i^{(f)}$ can result due to director field distortion. Following Meyer[13], $P_i^{(f)}$ is assumed to be a linear function of director gradients in the principal axes frame

$$P_i^{(f)} = e_{ijk}\alpha_{jk} \tag{19}$$

The tensor e_{ijk} has 27 components, all of which enter the picture. In frame indifferent notation (see Appendix 3)

$$P_i^{(f)} = \sum_a a_i(e_{aaa}I_{aab} + e_{aab}I_{aac} + e_{aac}I_{abc} + e_{aba}I_{bba} + e_{abb}I_{bbc} + e_{abc}I_{bca} + e_{aca}I_{cab} + e_{acb}I_{cca} + e_{acc}I_{ccb}) \tag{20}$$

where the e_{xyz} are linear combinations of the tensor components e_{ijk}. When flexoelectricity is present, $P_i^{(f)}$ has to be added to the dielectric polarisation for determining the total displacement and electric free energy in the presence of electric field induced director distortion.

B. Ericksen–Leslie approach

A general tensorial expansion of quantities in terms of independent variables is adopted here. The expressions for F and $P_i^{(f)}$ so derived are found to be formally identical to Eqs. (14) and (20) respectively. However, the viscous parts of σ_{ji} and g_i^a (CP) are found to be some what different. Using Eqs. (A4) and (B3) from the Appendices 1 and 2,

$$\begin{aligned}\sigma'_{ji} = \sum_a \big[& a_j a_i(\alpha_{aaaa}A_a + \alpha_{aaab}A_b + \alpha_{aaac}A_c + \alpha_{aaba}B_b + \alpha_{aabb}B_c \\ & + \alpha_{aabc}C_c + \alpha_{aaca}N_l a_l + \alpha_{aacb}N_l b_l + \alpha_{aacc}N_l c_l) \\ & + a_j b_i(\alpha_{abaa}A_a + \alpha_{abab}A_b + \alpha_{abac}A_c + \alpha_{abba}B_b + \alpha_{abbb}B_c \\ & + \alpha_{abbc}C_c + \alpha_{abca}N_l a_l + \alpha_{abcb}N_l b_l + \alpha_{abcc}N_l c_l) \\ & + a_j c_i(\alpha_{acaa}A_a + \alpha_{acab}A_b + \alpha_{acac}A_c + \alpha_{acba}B_b + \alpha_{acbb}B_c \\ & + \alpha_{acbc}C_c + \alpha_{acca}N_l a_l + \alpha_{accb}N_l b_l + \alpha_{accc}N_l c_l)\big],\end{aligned}$$

$$g_i^{a'} = a_i(\beta_{aaa}A_a + \beta_{aab}A_b + \beta_{aac}A_c + \beta_{aba}B_b + \beta_{abb}B_c + \beta_{abc}C_c + \beta_{aca}N_l a_l + \beta_{acb}N_l b_l + \beta_{acc}N_l c_l) + b_i(\delta_{aaa}A_a + \delta_{aab}A_b + \delta_{aac}A_c + \delta_{aba}B_b + \delta_{abb}B_c + \delta_{abc}C_c + \delta_{aca}N_l a_l + \delta_{acb}N_l b_l + \delta_{acc}N_l c_l) + c_i(\tau_{aaa}A_a + \tau_{aab}A_b + \tau_{aac}A_c + \tau_{aba}B_b + \tau_{abb}B_c + \tau_{abc}C_c + \tau_{aca}N_l a_l + \tau_{acb}N_l b_l + \tau_{acc}N_l c_l) \quad \text{(CP)} \quad (21)$$

The conservation of angular momentum (Eq. 7) shows that the viscous torque is determined by certain combinations of the coefficients δ, τ, which in turn are determined by combinations of α, the viscosities:

$$\alpha_{abaa} - \alpha_{baca} = \delta_{aaa} - \tau_{bca}; \qquad \alpha_{abab} - \alpha_{baba} = \delta_{aab} - \tau_{bba};$$

$$\alpha_{abac} - \alpha_{bacc} = \delta_{aac} - \tau_{bcc}; \qquad \alpha_{abba} - \alpha_{babb} = \delta_{aba} - \tau_{bbb};$$

$$\alpha_{abbb} - \alpha_{babc} = \delta_{abb} - \tau_{bbc}; \qquad \alpha_{abbc} - \alpha_{bacb} = \delta_{abc} - \tau_{bcb};$$

$$\alpha_{abca} - \alpha_{baaa} = \delta_{aca} - \tau_{baa}; \qquad \alpha_{abcb} - \alpha_{baab} = \delta_{acb} - \tau_{bab};$$

$$\alpha_{abcc} - \alpha_{baac} = \delta_{acc} - \tau_{bac}. \quad \text{(CP)} \quad (22)$$

A tensorial expansion thus results in an expression for σ'_{ji} which is described by 81 viscosity coefficients α. Applying Onsager's principle[3], one finds 18 relations which are satisfied by the α

$$a_c X_1 + b_c X_2 + c_c X_3 = \alpha_{baba} - \alpha_{abab};$$

$$a_a X_1 + a_b X_2 + a_c X_3 = \alpha_{cbaa} - \alpha_{bcba};$$

$$b_a X_1 + b_b X_2 + b_c X_3 = \alpha_{acab} - \alpha_{caaa};$$

$$c_a \alpha_{aaca} + c_b \alpha_{aacb} + c_c \alpha_{aacc} = \alpha_{baca} - \alpha_{abaa}; \quad (23)$$

$$a_a \alpha_{aaca} + a_b \alpha_{aacb} + a_c \alpha_{aacc} = \alpha_{cbac} - \alpha_{bcca};$$

$$b_a \alpha_{aaca} + b_b \alpha_{aacb} + b_c \alpha_{aacc} = \alpha_{acaa} - \alpha_{caac}; \quad \text{(CP)}$$

$$X_1 = \alpha_{abca} + \alpha_{baaa}; \qquad X_2 = \alpha_{abcb} + \alpha_{baab}; \qquad X_3 = \alpha_{abcc} + \alpha_{baac}.$$

Even with Eq. (23), full accord between Eqs. (18) and (21) is not obtained. For bringing Eq. (21) into the form of Eq. (18), the procedure outlined in ref. 3 is followed. Using Eqs. (21), (22) and (6) the entropy generation or equivalently φ, is written down. The coefficients are linear combinations of the α. Now, σ'_{ji} is calculated from Eq. (17) and equated to that given by Eq. (21), term by term. This results in all the α's being determined by the η's and the γ's of Eq. (18), and Eq. (21) can now be cast into the form given by Eq. (18). These equations which determine the α's include the Onsager relations (Eq. 23) and can be written as

$$\alpha_{aaaa} = \eta_{aaaa}; \qquad 2\alpha_{aacb} = \gamma_{baa}; \qquad 2\alpha_{aacc} = \gamma_{caa};$$

$$\alpha_{aaab} = \alpha_{abaa} + \alpha_{baca} = 2\eta_{aaab};$$

$$\alpha_{aaba} = \alpha_{bbca} = \eta_{aabb}; \qquad 2(\alpha_{baca} - \alpha_{abaa}) = a_c\gamma_{aaa} + b_c\gamma_{baa} + c_c\gamma_{caa};$$

$$\alpha_{aaac} = \alpha_{acaa} + \alpha_{caac} = 2\eta_{aaca}; \qquad 2\alpha_{aaca} = \gamma_{aaa};$$

$$2(\alpha_{acaa} - \alpha_{caac}) = b_a\gamma_{aaa} + b_b\gamma_{baa} + b_c\gamma_{caa}; \qquad \alpha_{abab} + \alpha_{baba} = 4\eta_{abab};$$

$$\alpha_{baba} - \alpha_{abab} = a_c\gamma_{aab} + b_c\gamma_{bab} + c_c\gamma_{cab};$$

$$\alpha_{aabb} = \alpha_{bcca} + \alpha_{cbac} = 2\eta_{aabc};$$

$$2(\alpha_{cbac} - \alpha_{bcca}) = a_a\gamma_{aaa} + a_b\gamma_{baa} + a_c\gamma_{caa}; \qquad (24)$$

$$\alpha_{abac} + \alpha_{bacc} = \alpha_{acab} + \alpha_{caaa} = 4\eta_{caab};$$

$$\alpha_{bacc} - \alpha_{abac} = c_a\gamma_{aca} + c_b\gamma_{bca} + c_c\gamma_{cca};$$

$$\alpha_{acab} - \alpha_{caaa} = b_a\gamma_{aab} + b_b\gamma_{bab} + b_c\gamma_{cab}; \qquad \alpha_{abca} + \alpha_{baaa} = \gamma_{aab};$$

$$\alpha_{baaa} - \alpha_{abca} = c_a\gamma_{aa} + c_b\gamma_{ab} + c_c\gamma_{ca}; \qquad \alpha_{abcb} + \alpha_{baab} = \gamma_{bab};$$

$$\alpha_{baab} - \alpha_{abcb} = c_a\gamma_{ab} + c_b\gamma_{bb} + c_c\gamma_{bc}; \qquad \alpha_{abcc} + \alpha_{baac} = \gamma_{cab};$$

$$\alpha_{baac} - \alpha_{abcc} = c_a\gamma_{ca} + c_b\gamma_{bc} + c_c\gamma_{cc}. \qquad \text{(CP)}$$

4. MONOCLINIC NEMATIC AND ORTHORHOMBIC NEMATIC

Let **a** be the diad axis, which is normal to **b** and **c**. Vectors **b** ai d **c** need not be perpendicular to one another. As the three director fields are no longer equivalent, the expressions of different physical quantities for a monoclinic nematic cease to be completely symmetrical with respect to **a**, **b** and **c** like those of a triclinic nematic (section 3) or an orthorhombic nematic[2]. In the principal axes frame, **a** and **b** of an aligned nematic can be made to coincide with the axes x_1 and x_2 respectively; **c** can lie in the x_2x_3 plane. A two-fold axis of symmetry about x_1 reduces the number of existing and independent components of μ_{ijkl} (Eq. 10) from 45 to 25. There can now be only four divergence terms, viz. D_{aa} (CP) and D_{bc} (Eq. 13). Eq. (14) reduces to

$$\rho F = F_0 + \lambda(u_{jj})^2/2 + \left\{ \sum_a (k_a + \kappa_a u_{jj}) I_{abc} + (l_c + \lambda_c u_{jj}) I_{cca} + (m_a + \mu_a u_{jj}) I_{bba} \right\}$$

$$+ \sum_a \left[k_{ab} I_{aab}^2 + k_{ac} I_{aac}^2 + k_{aa} I_{abc}^2 + 2c_{ab} I_{aac} I_{bbc} + 2k_{0a} D_{aa} \right]/2$$

$$+ c_{aa} I_{aab} I_{aac} + c_{ac} I_{cca} I_{cab} + l_{ab} I_{aab} I_{bbc} + l_{ac} I_{cca} I_{abc} + l_{bb} I_{bbc} I_{ccb}$$

$$+ m_{ac} I_{ccb} I_{aac} + m_{bc} I_{bba} I_{bca} + m_{cc} I_{cca} I_{bca} + p_{bb} I_{bba} I_{cab} + l_{0b} D_{bc} . \quad (25)$$

A monoclinic cholesteric will have the five additional linear terms { } which are absent for a monoclinic nematic. The elastic constants k, l, m, p are linear combinations of the existing tensor components of μ, with coefficients which now depend upon $\mathbf{b} \cdot \mathbf{c}$ and $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$. It is easy to show that the rest of the elastic constants in Eq. (14) vanish identically when **a** becomes a diad axis.

Turning now to the dissipative function (Eq. 15) η has 13, $\gamma^{(1)}$ 4 and $\gamma^{(2)}$, 8 surviving components and thus a monoclinic nematic is described by 25 viscosity coefficients, which is the same as the number of elastic constants and surface terms. The viscous stress is given by

$$\sigma'_{ji} = \sum_a a_i a_j [2\eta_{aaaa} A_a + 2\eta_{aabb} B_b + 2\eta_{aacc} C_c]/2$$
$$+ a_i a_j [4\eta_{aabc} B_c + \gamma_{aaa} N_k a_k]/2 + b_i b_j [4\eta_{bbbc} B_c + \gamma_{abb} N_k a_k]/2$$
$$+ c_i c_j [4\eta_{ccbc} B_c + \gamma_{acc} N_k a_k]/2$$
$$+ a_j b_i [\eta^-_{abab} A_b + \eta^-_{caab} A_c + \gamma^-_{bab} N_l b_l + \gamma^-_{cab} N_k c_k]/2$$
$$+ b_j c_i [\eta^-_{aabc} A_a + \eta^-_{bbbc} B_b + \eta^-_{ccbc} C_c + \eta^-_{bcbc} B_c + \gamma^-_{abc} N_l a_l]/2$$
$$+ c_j a_i [\eta_{caab} A_b + \eta^-_{caca} A_c + \gamma^-_{cca} N_l c_l + \gamma^-_{bca} N_k b_k]/2$$
$$+ a_i b_j [\eta^+_{abab} A_b + \eta^+_{caab} A_c + \gamma^+_{bab} N_l b_l + \gamma^+_{cab} N_k c_k]/2$$
$$+ b_i c_j [\eta^+_{aabc} A_a + \eta^+_{bbbc} B_b + \eta^+_{ccbc} C_c + \eta^+_{bcbc} B_c + \gamma^+_{abc} N_l a_l]/2$$
$$+ c_i a_j [\eta^+_{caab} A_b + \eta^+_{caca} A_c + \gamma^+_{cca} N_l c_l + \gamma^+_{bca} N_k b_k]/2,$$

$$\eta^\pm_{abab} = 4\eta_{abab} \pm (\gamma_{bab} b_c + \gamma_{cab} c_c), \qquad \eta^\pm_{caab} = 4\eta_{caab} \pm (\gamma_{bca} b_c + \gamma_{cca} c_c),$$
$$\gamma^\pm_{bab} = \gamma_{bab} \pm (\gamma_{bb} b_c + \gamma_{bc} c_c), \qquad \gamma^\pm_{cab} = \gamma_{cab} \pm (\gamma_{bc} b_c + \gamma_{cc} c_c),$$
$$\eta^\pm_{aabc} = 2\eta_{aabc} \pm \gamma_{aaa} a_a/2, \qquad \eta^\pm_{bbbc} = 2\eta_{bbbc} + \gamma_{abb} a_a/2,$$
$$\eta^\pm_{ccbc} = 2\eta_{ccbc} \pm \gamma_{acc} a_a/2, \qquad \eta^\pm_{bcbc} = 4\eta_{bcbc} \pm \gamma_{abc} a_a, \tag{26}$$
$$\gamma^\pm_{abc} = \gamma_{abc} \pm \gamma_{aa} a_a,$$
$$\eta^\pm_{caab} = 4\eta_{caab} \pm (\gamma_{bab} b_b + \gamma_{cab} b_c), \qquad \eta^\pm_{caca} = 4\eta_{caca} \pm (\gamma_{bca} b_b + \gamma_{cca} b_c),$$
$$\gamma^\pm_{bca} = \gamma_{bca} \pm (\gamma_{bb} b_b + \gamma_{bc} b_c), \qquad \gamma^\pm_{cca} = \gamma_{cca} \pm (\gamma_{bc} b_b + \gamma_{cc} b_c)$$

The ELA can be used along the lines of section 3. It is of course necessary to restrict terms which allow a diad axis about **a**. σ'_{ji} is found to depend on 41 αs. The Onsager relation yields 8 equations relating some of the 41 αs. The DFA results in 16 relations which include the eight Onsager relations. With these equations the 41 αs can be defined in terms of the ηs and γs of Eq. (26). One essentially gets subsets of Eqs. (21)–(24).

For the flexoelectric polarisation, the surviving components of e_{ijk}

(Eq. 19) number 13. For a monoclinic nematic,

$$P_i^{(f)} = \sum_a a_i(e_{aba}I_{bba} + e_{acb}I_{cca}) + c_i(e_{caa}I_{aab} + e_{cca}I_{ccb})$$
$$+ a_i(e_{aac}I_{abc} + e_{abc}I_{bca} + e_{aca}I_{cab}) + b_i(e_{baa}I_{aac} + e_{bbb}I_{bbc}) \tag{27}$$

A general tensorial expansion restricted by a diad axis about **a** leads to a formally identical expression as Eq. (27).

For the orthorhombic class of biaxial symmetry, Eq. (25) reduces to the expression for F given by Saupe;[2] Eq. (26) goes over to the expression for σ'_{ji} for a compressible orthorhombic nematic[2]. The flexoelectric polarisation $P_i^{(f)}$ is described by six constants

$$P_i^{(f)} = \sum_a a_i(e_{aba}I_{bba} + e_{acb}I_{cca}) \tag{28}$$

If uniaxial symmetry is imposed about one of the directors, say **a**, Eq. (28) becomes

$$P_i^{(f)} = e_1 a_i a_{k,k} + e_3 a_k a_{i,k} \tag{29}$$

which is Meyer's expression for a uniaxial nematic[13] which is described by director **a**. (e_1 and e_3 are the flexoelectric constants of a uniaxial nematic; also, $e_{bcb} = e_{cba} = 0$, $e_{aba} = e_{acb} = -e_1$, $e_{cac} = e_{bac} = e_3$).

5. INCOMPRESSIBLE FLUID

For a triclinic nematic, using Eq. (A4),

$$d_{kk} = \sum_a (h_{aa}A_a + 2h_{ab}A_b) \tag{30}$$

Using Eqs. (30) and (A4), it is possible to absorb the six viscosity coefficients η_{aaaa}, η_{abab} (CP) in linear combinations with other terms. The viscous stress can now be shown to be given by Eq. (18) with the restriction that the terms $a_i a_j \eta_{aaaa}$ (CP) will not occur and

$$\begin{aligned}
&\eta_{aaab} \to \eta_{aaab} - h_{ab}\eta_{aaaa}/2h_{aa} - h_{aa}\eta_{abab}/2h_{ab},\\
&\eta_{aaca} \to \eta_{aaca} - h_{ca}\eta_{aaaa}/2h_{aa} - h_{aa}\eta_{caca}/2h_{ca},\\
&\eta_{aabb} \to \eta_{aabb} - h_{bb}\eta_{aaaa}/2h_{aa} - h_{aa}\eta_{bbbb}/2h_{bb},\\
&\eta_{aabc} \to \eta_{aabc} - h_{bc}\eta_{aaaa}/2h_{aa} - h_{aa}\eta_{bcba}/2h_{bc},\\
&\eta^{\pm}_{abab} \to \eta^{\pm}_{abab} - 4\eta_{abab} \qquad \text{(CP)}
\end{aligned} \tag{31}$$

An incompressible triclinic will thus be described by 39 viscosity coefficients which is the same as the number of elastic constants.

As the elimination of η_{abab} (CP) involves division by the quantities h_{ab} (CP) two of which are zero for a monoclinic nematic, the expression of σ'_{ji} for an incompressible monoclinic nematic has to be obtained from that for a compressible monoclinic nematic and cannot be directly deduced from the expression for an incompressible triclinic nematic. For a monoclinic nematic, using Eq. (A7) one finds that

$$d_{kk} = A_a + h_{bb}(B_b + C_c) + 2h_{bc}B_c \tag{32}$$

With Eqs. (32) and (A7) the viscosity coefficients η_{aaaa} (CP) and η_{bcbc} can be eliminated from the expression for the viscous stress. σ'_{ji} is now given by Eq. (26) subject to the restrictions that the terms $a_i a_j A_a$ (CP) do not appear and

$$\begin{aligned}
&\eta_{aabb} \to \eta_{aabb} - (h_{bb}\eta_{aaaa} + \eta_{bbbb}/h_{bb})/2,\\
&\eta_{bbcc} \to \eta_{bbcc} - (\eta_{bbbb} + \eta_{cccc})/2,\\
&\eta_{ccaa} \to \eta_{ccaa} - (h_{bb}\eta_{aaaa} + \eta_{cccc}/h_{bb})/2,\\
&\eta_{aabc} \to \eta_{aabc} - (h_{bc}\eta_{aaaa} + \eta_{bcbc}/h_{bc})/2,\\
&\eta_{bbbc} \to \eta_{bbbc} - (h_{bc}\eta_{bbbb}/h_{bb} + h_{bb}\eta_{bcbc}/h_{bc})/2,\\
&\eta_{ccbc} \to \eta_{ccbc} - (h_{bc}\eta_{cccc}/h_{bb} + h_{bb}\eta_{bcbc}/h_{bc})/2,\\
&\eta^{\pm}_{bcbc} \to \eta^{\pm}_{bcbc} - 4\eta_{bcbc}
\end{aligned} \tag{33}$$

An incompressible monoclinic nematic will thus be described by 21 viscosity coefficients which is the same as the number of elastic constants.

It is again not possible to deduce the expression of σ'_{ji} for an incompressible orthorhombic nematic from that of σ'_{ji} for an incompressible monoclinic or triclinic nematic, for obvious reasons. One can however start from Eq. (18) or from Eq. (26), reduce it to orthorhombic symmetry, use Eq. (A8) to get $d_{kk} = \sum_a A_a$, eliminate the three viscosity coefficients η_{aaaa} (CP) and obtain an expression which is identical to that given by Saupe.[2] It is possible to rewrite the expression (16) using Eq. (30) to obtain entropy generation in an incompressible triclinic. A similar expression can also be written for an incompressible monoclinic nematic. These have been left out.

Expressions for the external director body forces ρG_i^a (CP) can be derived by following Ericksen.[16] When the inhomogeneity of the medium brings about an inhomogeneity in the applied field, the field may also contribute towards the external body force ρf_i. Some simple examples of hydrostatic and hydrodynamic behaviour such as Freedericksz transition, back flow, rotating magnetic field and rotating sample, power spectrum of hydrodynamic fluctuations, simple examples of electrohydrodynamic instabilities and flexoelectric domains etc. are likely to be presented in the near future.[17,18]

APPENDIX 1. DERIVATION OF THE METRIC

Triclinic nematic

If $\mathbf{a}^0, \mathbf{b}^0, \mathbf{c}^0$ are three orthonormal vactors,

$$\sum_a a_i^0 a_j^0 = \delta_{ij} \tag{A1}$$

Let a triclinic nematic be described by three non-coplanar unit vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$. Without loss of generality, $\mathbf{a}$ can be chosen parallel to $\mathbf{a}^0$, and $\mathbf{b}$ to lie in the $(\mathbf{a}^0, \mathbf{b}^0)$ plane. In the $\mathbf{a}^0, \mathbf{b}^0, \mathbf{c}^0$ frame, using Eq. (8)

$$\mathbf{a} = \mathbf{a}^0, \qquad \mathbf{b} = d_1\mathbf{a}^0 + d_2\mathbf{b}^0, \qquad \mathbf{c} = e_1\mathbf{a}^0 + e_2\mathbf{b}^0 + e_3\mathbf{c}^0 \tag{A2}$$

Solving for $\mathbf{a}^0, \mathbf{b}^0, \mathbf{c}^0$, one gets

$$\begin{gathered}\mathbf{a}^0 = \mathbf{a}, \qquad \mathbf{b}^0 = f_1\mathbf{a} + f_2\mathbf{b}, \qquad \mathbf{c}^0 = g_1\mathbf{a} + g_2\mathbf{b} + g_3\mathbf{c}, \\ f_1 = -d_1/d_2, \qquad f_2 = 1/d_2, \\ g_1 = (e_2d_1 - e_1d_2)/e_3d_2, \qquad g_2 = -e_2/e_3d_2, \qquad g_3 = 1/e_3\end{gathered} \tag{A3}$$

Using Eqs. (A1) and (A3) it is possible to write

$$\delta_{ij} \sum_a \left[h_{aa} a_i a_j + h_{ab}(a_i b_j + a_j b_i) \right] \tag{A4}$$

$$\begin{gathered}h_{aa} = \left[e_3^2 + (e_2 d_1 - e_1 d_2)^2 \right] / e_3^2 d_2^2, \\ h_{ab} = \left[e_1 e_2 d_2 - d_1(e_2^2 + e_3^2) \right] / e_3^2 d_2^2, \\ h_{ca} = (e_2 d_1 - e_1 d_2)/e_3^2 d_2, \qquad h_{bb} = (e_2^2 + e_3^2)/e_3^2 d_2^2, \\ h_{bc} = -e_2/e_3^2 d_2, \qquad h_{cc} = 1/e_3^2\end{gathered} \tag{A5}$$

Monoclinic nematic

a is the diad axis, which is normal to **b** and **c**. Hence,

$$d_1 = e_1 = f_1 = g_1 = 0, \qquad d_2 = e_2^2 + e_3^2 = 1 = f_2, \\ g_2 = -e_2/e_3, \qquad g_3 = 1/e_3 \tag{A6}$$

$$\delta_{ij} = a_i a_j + h_{bb}(b_i b_j + c_i c_j) + h_{bc}(b_i c_j + b_j c_i), \\ h_{bb} = h_{cc} = 1/e_3^2, \qquad h_{bc} = -e_2/e_3^2 \tag{A7}$$

Orthorhombic nematic

All three vectors **a**, **b**, **c**, are mutually perpendicular to one another and all three are diad axes of symmetry. As compared to Eqs. (A6) and (A7) we further have $e_2 = 0$, $e_3 = 1$, and

$$\delta_{ij} = \sum_a a_i a_j \tag{A8}$$

Expressions on the right hand sides in Eqs. (A4), (A7) and (A8) indicate the form of a general symmetrical tensor for that particular class of biaxial symmetry. Material properties such as dielectric permittivity, thermal conductivity, electrical conductivity, diamagnetic susceptibility etc. are represented by symmetrical tensors. Thus the electrical conductivity tensor for a triclinic nematic

$$K_{ij}^{(e)} = \sum_a \left[K_{aa}^{(e)} a_i a_j + K_{ab}^{(e)} (a_i b_j + a_j b_i) \right] \tag{A9}$$

is described by six constants $K_{aa}^{(e)}, K_{ab}^{(e)}$ (CP). The dielectric tensor of a monoclinic nematic

$$\epsilon_{ij} = \sum_a \epsilon_{aa} a_i a_j + \epsilon_{bc}(b_i c_j + b_j c_i) \tag{A10}$$

depends upon four dielectric constants ϵ_{aa} (CP) and ϵ_{bc}. The diamagnetic susceptibility of an orthorhombic nematic

$$\chi_{ij} = \sum_a \chi_{aa} a_i a_j \tag{A11}$$

is described by three susceptibilities χ_{aa} (CP) and is a diagonal tensor in the principal axes frame. For the triclinic and monoclinic classes, it should be possible to transform to principal axes and convert a symmetric matrix into diagonal form. It is not known whether this will lead to a simplification of the differential equations.

APPENDIX 2. VELOCITY OF ROTATION N OF THE DIRECTOR FIELD RELATIVE TO THE FLUID

Triclinic nematic

It is known[2,3] that the velocity of rotation of the orthornormal triad $\mathbf{a}^0, \mathbf{b}^0, \mathbf{c}^0$ relative to the fluid which is described by the velocity field $v_i(x_k)$ is

$$\mathbf{N} = \sum_a (\mathbf{N}^{b0} \cdot \mathbf{c}^0)\mathbf{a}^0 \quad \text{or} \quad N_i = \sum_a a_i^0(\dot{b}_k^0 - \omega_{kl} b_l^0) c_k^0 \tag{B1}$$

For a triclinic nematic which is described by the non-coplanar director triad, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and a velocity field $v_i\ (x_k)$ the same construction as in Appendix 1 is effected. The triad $\mathbf{a}^0, \mathbf{b}^0, \mathbf{c}^0$ is assumed to be rigidly fixed to the frame $\mathbf{a}, \mathbf{b}, \mathbf{c}$. The rotation velocity of $\mathbf{a}, \mathbf{b}, \mathbf{c}$, can be equally well described by Eq. (B1). Using Eqs. (A2), (A3), and (B1) one finds that

$$N_i = \sum_a (N_k^b c_k) a_i / (\mathbf{a} \cdot \mathbf{b} \times hc) \tag{B2}$$

In terms of N_k, the N_i^a (CP) can be expressed as

$$N_i^a = a_i[c_a N_l b_l - a_b N_l c_l] + b_i[a_a N_l c_l - c_a N_l a_l]$$
$$+ c_i[a_b N_l a_l - a_a N_l b_l] \qquad \text{(CP)}$$
$$a_b = \mathbf{a} \cdot \mathbf{b}/V = d_1/e_3 d_2 = b_a,$$
$$b_c = \mathbf{b} \cdot \mathbf{c}/V = (e_1 d_1 + e_2 d_2)/e_3 d_2 = c_b, \tag{B3}$$
$$c_a = \mathbf{c} \cdot \mathbf{a}/V = e_1/e_3 d_2 = a_c, \qquad a_a = b_b = c_c = 1/V = 1/e_3 d_2,$$
$$V = \mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$$

Monoclinic nematic

$\mathbf{a}$ is the diad axis. $\mathbf{N}$ is given by Eq. (B2) with $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = e_3$. The N_i^a (CP) are given by

$$N_i^a = b_i N_l c_l - c_i N_l b_l a_a,$$
$$N_i^b = -b_c N_l a_l b_i + b_b N_l a_l c_i + a_i[b_c N_l b_l - b_b N_l c_l], \tag{B4}$$
$$N_i^c = c_i b_c N_l a_l + a_i[c_c N_l b_l - b_c N_l c_l] - b_i c_c N_l a_l,$$
$$b_c = e_2/e_3, \qquad a_a = b_b = c_c = 1/e_3$$

Orthorhombic nematic

$\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{c}=\mathbf{c}\cdot\mathbf{a}=0$; $a_a=b_b=c_c=1$; $\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}=1$. Eq. (B1) represents $\mathbf{N}$ and

$$N_i^a = b_i N_l c_l - c_i N_l b_1 \qquad \text{(CP)} \qquad \text{(B5)}$$

If Ω_{kl} is the antisymmetrical tensor which describes the rotation of the director field relative to the fluid,[3]

$$N_i = \Omega_{kl} \sum_a b_l c_k a_i / (\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}) \qquad \text{(B6)}$$

APPENDIX 3

In order to write F and σ'_{ji} in frame indifferent notation, the following transformations are used:

Triclinic nematic

$$\alpha_{11}\to f_3 I_{aab} + g_3 J_{aa}, \qquad \alpha_{21}\to J_{bb}, \qquad \alpha_{22}\to f_1 J_{bb} + f_2 J_{bc},$$

$$\alpha_{31}\to f_2 I_{aab}, \qquad \alpha_{32}\to f_2(f_1 I_{aab} - f_2 I_{bba}), \qquad \alpha_{33}\to f_2 J_{ac},$$

$$\alpha_{23}\to g_1 J_{bb} + g_2 J_{bc} + g_3(g_3 I_{cca} - g_2 I_{cab}),$$

$$\alpha_{12}\to f_3 J_{ab} + g_3(f_1^2 I_{aac} + f_2^2 I_{bbc}) + f_1 f_2 g_3(I_{abc} - I_{bca}),$$

$$\alpha_{13}\to f_3 J_{ac} - g_3^2(f_1 I_{cca} + f_2 I_{ccb}) + g_2 g_3(f_2 I_{bbc} - f_1 I_{bca}) + g_1 g_3 J_{aa},$$

$$f_3 = f_1 g_2 - f_2 g_1, \qquad J_{aa} = f_1 I_{aac} + f_2 I_{abc}, \qquad J_{ab} = f_1 I_{aab} - f_2 I_{bba},$$

$$J_{ac} = g_1 I_{aab} - g_2 I_{bba} + g_3 I_{cab}, \qquad J_{bb} = -g_2 I_{aab} - g_3 I_{aac}, \qquad \text{(C1)}$$

$$J_{bc} = g_2 I_{bba} + g_3 I_{bca}, \qquad \dot\alpha_1 - \dot\beta_1 \to N_l a_l,$$

$$\dot\alpha_2 - \dot\beta_2 \to f_1 N_l a_l + f_2 N_l b_l, \qquad \dot\alpha_3 - \dot\beta_3 \to g_1 N_l a_l + g_2 N_l b_l + g_3 N_l c_l,$$

$$d_{11}\to A_a, \qquad d_{12}\to f_1 A_a + f_2 A_b, \qquad d_{13}\to g_1 A_a + g_2 A_b + g_3 A_c,$$

$$d_{22}\to f_1^2 A_a + 2 f_1 f_2 A_b + f_2^2 B_b,$$

$$d_{23}\to f_1 g_1 A_a + (f_1 g_2 + f_2 g_1) A_b + f_1 g_3 A_c + f_2 g_3 B_c + f_2 g_2 B_b,$$

$$d_{33}\to g_1^2 A_a + 2 g_1 g_2 A_b + 2 g_1 g_3 A_c + 2 g_2 g_3 B_c + g_2^2 B_b + g_3^2 C_c$$

Monoclinic nematic

$d_1 = e_1 = f_1 = g_1 = 0$, $d_2 = f_2 = 1$. From Eq. (C1) one gets

$$\alpha_{11} \to g_3 I_{abc}, \quad \alpha_{12} \to g_3 I_{bbc}, \quad \alpha_{13} \to -g_3^2 I_{ccb} + g_2 g_3 I_{bbc},$$
$$\alpha_{21} \to -g_2 I_{aab} - g_3 I_{aac}, \quad \alpha_{22} \to g_2 I_{bba} + g_3 I_{bca},$$
$$\alpha_{23} \to g_2(g_2 I_{bba} + g_3 I_{bca}) + g_3(g_3 I_{cca} - g_2 I_{cab}),$$
$$\alpha_{31} \to I_{aab}, \quad \alpha_{32} \to -I_{bba}, \quad \alpha_{33} \to g_3 I_{cab} - g_2 I_{bba},$$
$$\dot{\alpha}_1 - \dot{\beta}_1 \to N_l a_l, \quad \dot{\alpha}_2 - \dot{\beta}_2 \to N_l b_l, \tag{C2}$$
$$\dot{\alpha}_3 - \dot{\beta}_3 \to g_2 N_l b_l + g_3 N_l c_l, \quad d_{11} \to A_a, \quad d_{12} \to A_b,$$
$$d_{13} \to g_2 A_b + g_3 A_c, \quad d_{22} \to B_b,$$
$$d_{23} \to g_3 B_c + g_2 B_b, \quad d_{33} \to g_2^2 B_b + 2 g_2 g_3 B_c + g_3^2 C_c$$

Orthorhombic nematic

The transformations can be deduced from Eq. (C2) with the further restriction that $e_2 = g_3 = 0$, $e_3 = g_3 = 1$. Then,

$$\alpha_{11} \to I_{abc}, \quad \alpha_{12} \to I_{bbc}, \quad \alpha_{13} \to -I_{ccb},$$
$$\alpha_{21} \to -I_{aac}, \quad \alpha_{22} \to I_{bca}, \quad \alpha_{23} \to I_{cca},$$
$$\alpha_{31} \to I_{aab}, \quad \alpha_{32} \to -I_{bba}, \quad \alpha_{33} \to I_{cab}, \tag{C3}$$
$$\dot{\alpha}_1 - \dot{\beta}_1 \to N_l a_l, \quad \dot{\alpha}_2 - \dot{\beta}_2 \to N_l b_l, \quad \dot{\alpha}_3 - \dot{\beta}_3 \to N_l c_l, \quad d_{11} \to A_a,$$
$$d_{12} \to A_b, \quad d_{13} \to A_c, \quad d_{22} \to B_b, \quad d_{23} \to B_c, \quad d_{33} \to C_c.$$

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